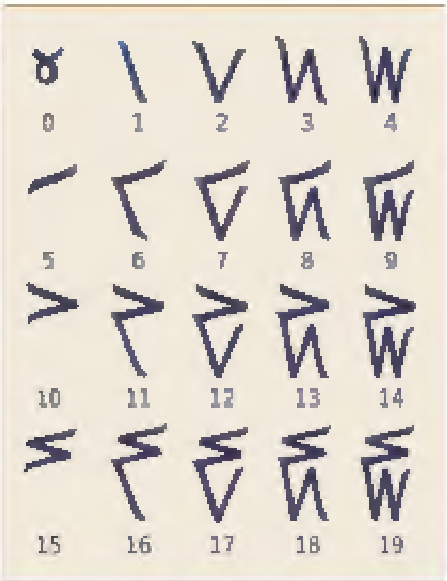
Kaktovik numerals

The **Kaktovik numerals** or **Kaktovik Iñupiaq numerals**[1] are a base-20 system of numerical digits created by Alaskan Iñupiat. They are visually iconic, with shapes that indicate the number being represented.



The 20 digits of the Kaktovik system

You may need rendering support to display the uncommon Unicode characters in this article correctly.

The Iñupiaq language has a base-20 numeral system, as do the other Eskimo–Aleut languages of Alaska and Canada (and formerly Greenland). Arabic numerals, which were designed for a base-10 system, are inadequate for Iñupiaq and other Inuit languages. To remedy this problem, students in Kaktovik, Alaska, invented a base-20 numeral notation in 1994, which has spread among the Alaskan Iñupiat and has been considered for use in Canada.

System

Iñupiaq, like other Inuit languages, has a base-20 counting system with a sub-base of 5 (a *quinary-vigesimal* system). That is, quantities are counted in scores (as in Welsh and French *quatre-vingts* 'eighty'), with intermediate numerals for 5, 10, and 15. Thus 78 is identified as *three score fifteen-three*.[2]

The Kaktovik digits graphically reflect the lexical structure of the Iñupiaq numbering system.[3]

The twenty digits

0	𝋀	*kisitchisaġvik*	**5**	𝋅	*tallimat*	**10**	𝋊	*qulit*	**15**	𝋏	*akimiaq*
1	𝋁	*atausiq*	**6**	𝋆	*itchaksrat*	**11**	𝋋	*qulit atausiq*	**16**	𝋐	*akimiaq atausiq*
2	𝋂	*malġuk*	**7**	𝋇	*tallimat malġuk*	**12**	𝋌	*qulit malġuk*	**17**	𝋑	*akimiaq malġuk*
3	𝋃	*piŋasut*	**8**	𝋈	*tallimat piŋasut*	**13**	𝋍	*qulit piŋasut*	**18**	𝋒	*akimiaq piŋasut*
4	𝋄	*sisamat*	**9**	𝋉	*quliŋŋuġutaiḷaq*	**14**	𝋎	*akimiaġutaiḷaq*	**19**	𝋓	*iñuiññaġutaiḷaq*

Larger numbers are composed of these digits in a positional notation:

Decimal	**Vigesimal**	
Arabic	**Arabic**	**Kaktovik**
20	10_{20}	𝋁𝋀
40	20_{20}	𝋂𝋀
400	100_{20}	𝋁𝋀𝋀
800	200_{20}	𝋂𝋀𝋀

Values

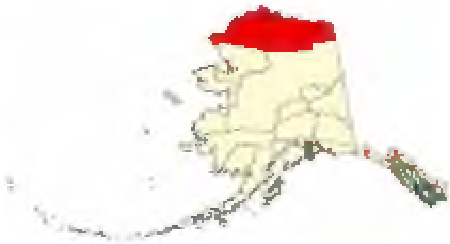
In the following table are the decimal values of the Kaktovik digits up to three places to the left and to the right of the units' place.[3]

Decimal values of Kaktovik numbers

n	$n \times 20^3$	$n \times 20^2$	$n \times 20^1$	$n \times 20^0$	$n \times 20^{-1}$	$n \times 20^{-2}$	$n \times 20^{-3}$
1	𝋁,𝋀𝋀𝋀 8,000	𝋁𝋀𝋀 400	𝋁𝋀 20	𝋁 1	𝋀.𝋁 0.05	𝋀.𝋀𝋁 0.0025	𝋀.𝋀𝋀𝋁 0.000 125
2	𝋂,𝋀𝋀𝋀 16,000	𝋂𝋀𝋀 800	𝋂𝋀 40	𝋂 2	𝋀.𝋂 0.1	𝋀.𝋀𝋂 0.005	𝋀.𝋀𝋀𝋂 0.000 25
3	𝋃,𝋀𝋀𝋀 24,000	𝋃𝋀𝋀 1,200	𝋃𝋀 60	𝋃 3	𝋀.𝋃 0.15	𝋀.𝋀𝋃 0.0075	𝋀.𝋀𝋀𝋃 0.000 375
4	𝋄,𝋀𝋀𝋀 32,000	𝋄𝋀𝋀 1,600	𝋄𝋀 80	𝋄 4	𝋀.𝋄 0.2	𝋀.𝋀𝋄 0.01	𝋀.𝋀𝋀𝋄 0.000 5
5	𝋅,𝋀𝋀𝋀 40,000	𝋅𝋀𝋀 2,000	𝋅𝋀 100	𝋅 5	𝋀.𝋅 0.25	𝋀.𝋀𝋅 0.0125	𝋀.𝋀𝋀𝋅 0.000 625
6	𝋆,𝋀𝋀𝋀 48,000	𝋆𝋀𝋀 2,400	𝋆𝋀 120	𝋆 6	𝋀.𝋆 0.3	𝋀.𝋀𝋆 0.015	𝋀.𝋀𝋀𝋆 0.000 75
7	𝋇,𝋀𝋀𝋀 56,000	𝋇𝋀𝋀 2,800	𝋇𝋀 140	𝋇 7	𝋀.𝋇 0.35	𝋀.𝋀𝋇 0.0175	𝋀.𝋀𝋀𝋇 0.000 875
8	𝋈,𝋀𝋀𝋀 64,000	𝋈𝋀𝋀 3,200	𝋈𝋀 160	𝋈 8	𝋀.𝋈 0.4	𝋀.𝋀𝋈 0.02	𝋀.𝋀𝋀𝋈 0.001
9	𝋉,𝋀𝋀𝋀 72,000	𝋉𝋀𝋀 3,600	𝋉𝋀 180	𝋉 9	𝋀.𝋉 0.45	𝋀.𝋀𝋉 0.0225	𝋀.𝋀𝋀𝋉 0.001 125
10	𝋊,𝋀𝋀𝋀 80,000	𝋊𝋀𝋀 4,000	𝋊𝋀 200	𝋊 10	𝋀.𝋊 0.5	𝋀.𝋀𝋊 0.025	𝋀.𝋀𝋀𝋊 0.001 25
11	𝋋,𝋀𝋀𝋀 88,000	𝋋𝋀𝋀 4,400	𝋋𝋀 220	𝋋 11	𝋀.𝋋 0.55	𝋀.𝋀𝋋 0.0275	𝋀.𝋀𝋀𝋋 0.001 375
12	𝋌,𝋀𝋀𝋀 96,000	𝋌𝋀𝋀 4,800	𝋌𝋀 240	𝋌 12	𝋀.𝋌 0.6	𝋀.𝋀𝋌 0.03	𝋀.𝋀𝋀𝋌 0.001 5
13	𝋍,𝋀𝋀𝋀 104,000	𝋍𝋀𝋀 5,200	𝋍𝋀 260	𝋍 13	𝋀.𝋍 0.65	𝋀.𝋀𝋍 0.0325	𝋀.𝋀𝋀𝋍 0.001 625
14	𝋎,𝋀𝋀𝋀 112,000	𝋎𝋀𝋀 5,600	𝋎𝋀 280	𝋎 14	𝋀.𝋎 0.7	𝋀.𝋀𝋎 0.035	𝋀.𝋀𝋀𝋎 0.001 75
15	𝋏,𝋀𝋀𝋀 120,000	𝋏𝋀𝋀 6,000	𝋏𝋀 300	𝋏 15	𝋀.𝋏 0.75	𝋀.𝋀𝋏 0.0375	𝋀.𝋀𝋀𝋏 0.001 875
16	𝋐,𝋀𝋀𝋀 128,000	𝋐𝋀𝋀 6,400	𝋐𝋀 320	𝋐 16	𝋀.𝋐 0.8	𝋀.𝋀𝋐 0.04	𝋀.𝋀𝋀𝋐 0.002
17	𝋑,𝋀𝋀𝋀 136,000	𝋑𝋀𝋀 6,800	𝋑𝋀 340	𝋑 17	𝋀.𝋑 0.85	𝋀.𝋀𝋑 0.0425	𝋀.𝋀𝋀𝋑 0.002 125
18	𝋒,𝋀𝋀𝋀 144,000	𝋒𝋀𝋀 7,200	𝋒𝋀 360	𝋒 18	𝋀.𝋒 0.9	𝋀.𝋀𝋒 0.045	𝋀.𝋀𝋀𝋒 0.002 25

19	𝋓·𝋀𝋀𝋀 152,000	𝋓𝋀𝋀 7,600	𝋓𝋀 380	𝋓 19	𝋀.𝋓 0.95	𝋀.𝋀𝋓 0.0475	𝋀.𝋀𝋀𝋓 0.002 375

Origin



Map of Alaska highlighting North Slope Borough, part of Iñupiat Nunaat

The numerals began as an enrichment activity in 1994, when, during a math class exploring binary numbers at Harold Kaveolook middle school on Barter Island Kaktovik, Alaska,[4] students noted that their language used a base-20 system.

They found that, when they tried to write numbers or do arithmetic with Arabic numerals, they did not have enough symbols to represent the Iñupiaq numbers.[5]

They first addressed this lack by creating ten extra symbols, but found these were difficult to remember. The small middle school had only nine students so the entire school was able to work together to create a base-20 notation. Their teacher, William Bartley, guided them.[5]

After brainstorming, the students came up with several qualities that an ideal system would have:

1. Visual simplicity: The symbols should be "easy to remember."
2. Iconicity: There should be a "clear relationship between the symbols and their meanings."
3. Efficiency: It should be "easy to write" the symbols, and they should be able to be "written quickly" without lifting the pencil from the paper.
4. Distinctiveness: They should "look very different from Arabic numerals," so there would not be any confusion between notation in the two systems.
5. Aesthetics: They should be pleasing to look at.[5]

In base-20 positional notation, the number twenty is written with the digit for 1 followed by the digit for 0. The Iñupiaq language does not have a word for zero, and the students decided that the Kaktovik digit 0 should look like crossed arms, meaning that nothing was being counted.[5]

When the middle-school pupils began to teach their new system to younger students in the school, the younger students tended to squeeze the numbers down to fit inside the same-sized block. In this way, they created an iconic notation with the sub-base of 5 forming the upper part

of the digit, and the remainder forming the lower part. This proved visually helpful in doing arithmetic.[5]

Computation

Iñupiaq abacus designed for use with the Kaktovik numerals

Abacus

The students built base-20 abacuses in the school workshop.[4][5] These were initially intended to help the conversion from decimal to base-20 and vice versa, but the students found their design lent itself quite naturally to arithmetic in base-20. The upper section of their abacus had three beads in each column for the values of the sub-base of 5, and the lower section had four beads in each column for the remaining units.[5]

Arithmetic

An advantage the students discovered of their new system was that arithmetic was easier than with the Arabic numerals.[5] Adding two digits together would look like their sum. For example,

2 + 2 = 4

𝋂 + 𝋂 = 𝋄

It was even easier for subtraction: one could simply look at the number and remove the appropriate number of strokes to get the answer.[5] For example,

4 − 1 = 3

𝋄 − 𝋁 = 𝋃

Another advantage came in doing long division. The visual aspects and the sub-base of five made long division with large dividends almost as easy as short division, as it didn't require writing in sub-tables for multiplying and subtracting the intermediate steps.[4] The students could keep track of the strokes of the intermediate steps with colored pencils in an elaborated system of chunking.[5]

Examples of long division

Simple long division

$30{,}561_{10} \div 61_{10} = 501_{10}$

$3{,}\mathrm{G}81_{20} \div 31_{20} = 151_{20}$

𝋃𝋐𝋈𝋁 ÷ 𝋃𝋁 = 𝋁𝋅𝋁

(black) The divisor 𝋃𝋁 goes into the first two digits of the dividend one time, for a 𝋁 one in the quotient.

(red) 𝋃𝋁 fits into the next two digits once (if rotated), so the next digit in the quotient is a rotated one (that is, a 𝋅 five).

(blue) The last two digits are matched once for a final 𝋁 one in the quotient.

Long division with more chunking

$46{,}349{,}226_{10} \div 2{,}826_{10} = 16{,}401_{10}$

$\mathrm{E9D{,}D16}_{20} \div 716_{20} = 2{,}101_{20}$

𝋎𝋉𝋍𝋍𝋁𝋆 ÷ 𝋇𝋁𝋆 = 𝋂𝋁𝋀𝋁

(black and red) The divisor 𝋇𝋁𝋆 goes into the first three digits of the dividend twice (once in black and once in red), for a two in the quotient.

(blue) 𝋇𝋁𝋆 goes into the next three digits once, for a 𝋁 one in the quotient.

(grey) 𝋇𝋁𝋆 does not fit into the next three digits, for a 𝋀 zero in the quotient.

(green) 𝋇𝋁𝋆 fits into the remaining digits once, for a final 𝋁 one in the quotient.

A simplified multiplication table can be made by first finding the products of each base digit, then the products of the bases and the sub-bases, and finally the product of each sub-base:

×	𝋁 1	𝋂 2	𝋃 3	𝋄 4
1 𝋁	𝋁	𝋂	𝋃	𝋄
2 𝋂	𝋂	𝋄	𝋆	𝋈
3 𝋃	𝋃	𝋆	𝋉	𝋌
4 𝋄	𝋄	𝋈	𝋌	𝋐

×	𝋁 1	𝋂 2	𝋃 3	𝋄 4
5 𝋅	𝋅	𝋊	𝋏	𝋁𝋀
10 𝋊	𝋊	𝋁𝋀	𝋁𝋊	𝋂𝋀
15 𝋏	𝋏	𝋁𝋊	𝋂𝋅	𝋃𝋀

×	𝋅 5	𝋊 10	𝋏 15
5 𝋅	𝋁𝋅	𝋂𝋊	𝋃𝋏
10 𝋊	𝋂𝋊	𝋅𝋀	𝋇𝋊
15 𝋏	𝋃𝋏	𝋇𝋊	𝋋𝋅

These tables are functionally complete for multiplication operations using Kaktovik numerals, but for factors with both bases and sub-bases it is necessary to first disassociate them:

6 * 3 = 18

𝋆 * 𝋃 = (𝋁 * 𝋃) + (𝋅 * 𝋃) = 𝋒

In the above example the factor 𝋆 (6) is not found in the table, but its components, 𝋁 (1) and 𝋅 (5), are.

Legacy

The Kaktovik numerals have gained wide use among Alaskan Iñupiat. They have been introduced into language-immersion programs and have helped revive base-20 counting, which had been falling into disuse among the Iñupiat due to the prevalence of the base-10 system in English-medium schools.[4][5]

When the Kaktovik middle school students who invented the system were graduated to the high school in Barrow, Alaska (now renamed Utqiaġvik), in 1995, they took their invention with them.

They were permitted to teach it to students at the local middle school, and the local community Iḷisaġvik College added an Inuit mathematics course to its catalog.[5]

In 1996, the Commission on Inuit History Language and Culture officially adopted the numerals,[5] and in 1998 the Inuit Circumpolar Council in Canada recommended the development and use of the Kaktovik numerals in that country.[6]

Significance

Scores on the California Achievement Test in mathematics for the Kaktovik middle school improved dramatically in 1997 compared to previous years. Before the introduction of the new numerals, the average score had been in the 20th percentile; after their introduction, scores rose to above the national average. It is theorized that being able to work in both base-10 and base-20 might have comparable advantages to those bilingual students have from engaging in two ways of thinking about the world.[5]

The development of an indigenous numeral system helps to demonstrate to Alaskan-native students that math is embedded in their culture and language rather than being imparted by western culture. This is a shift from a previously commonly held view that mathematics was merely a necessity to get into a college or university. Non-native students can see a practical example of a different world view, a part of ethnomathematics.[7]

In Unicode

The Kaktovik numerals were added to the Unicode Standard in September, 2022, with the release of version 15.0. Several fonts support this block.

Kaktovik Numerals[1][2]

Official Unicode Consortium code chart (https://www.unicode.org/charts/PDF/U1D2C0.pdf) (PDF)

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
U+1D2Cx	□	□	□	□	□	□	□	□	□	□	□	□	□	□	□	□
U+1D2Dx	□	□	□	□												

Notes

1. ^ As of Unicode version 16.0
2. ^ Grey areas indicate non-assigned code points

Graphical display of Kaktovik Numerals

Official Unicode Consortium code chart (https://www.unicode.org/charts/PDF/U1D2C0.pdf) (PDF)

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
U+1D2Cx	𝋀	𝋁	𝋂	𝋃	𝋄	𝋅	𝋆	𝋇	𝋈	𝋉	𝋊	𝋋	𝋌	𝋍	𝋎	𝋏
U+1D2Dx	𝋐	𝋑	𝋒	𝋓												

See also

- Maya numerals, a quinary-vigesimal system from another American culture

References

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3. MacLean 2014, p. 832.
4. Bartley 1997.
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Further reading

- Tillinghast-Raby, Amory (June 2023). "A Number System Invented by Inuit Schoolchildren Will Make Its Silicon Valley Debut" (https://www.scientificamerican.com/article/a-number-system-invented-by-inuit-schoolchildren-will-make-its-silicon-valley-debut/) . *Scientific American Magazine*. Retrieved April 13, 2023.

External links

- Kaktovik–Arabic converter (https://www.dcode.fr/kaktovik-numerals) , using either images or Unicode
- Grunewald, Edgar (December 30, 2019). "Why These Are The Best Numbers!" (https://www.youtube.com/watch?v=EyS6FfczH0Q&t=120) . *YouTube*. Archived (https://ghostarchive.org/varchive/youtube/20211220/EyS6FfczH0Q) from the original on December 20, 2021. Retrieved December 30, 2019. The video demonstrates how long division is easier with visually intuitive digits like the Kaktovik ones; the illustrated problems were chosen to work out easily, as the problems in a child's introduction to arithmetic would be.
- Silva, Eduardo Marín; Miller, Kirk; Strand, Catherine (April 29, 2021). "Unicode request for Kaktovik numerals (L2/21-058R)" (https://www.unicode.org/L2/L2021/21058r-kaktovik-numerals.pdf) (PDF). *Unicode Technical Committee Document Registry*. Retrieved April 30, 2021.
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